$$
\boxed{\top_{0 \text{pic}} - 5 \text{e} + s}
$$



Vef: A set is a collection of elements (or objects).  $IF × is an element of x$ set S, then we write XES. read: "x is in S" If x is not an element of a set S, then we write  $x \notin S$ .<br>read: "x is not in S"  $EX: S = \{0, 10, -1\}$  $0\in 5$  $-$  |  $10E5$  $\begin{matrix} 0 & 10 \ 0 & 0 \end{matrix}$  $-1$   $\epsilon$  S  $245$  $-545$ 

Note: There is no ordering<br>on the elements of a set on the elements of a set. Thus ,  $\Big\}$  0 , 10,  $-19 = 210, -109$ for example. Also , sets cannot have duplicate entries. For example you can't have  $\{1, 0\}$  $14$  as a set

$$
EX:\nN = \{1, 2, 3, 4, 5, 6, 7, ... \} \leftarrow \frac{\text{natural}}{\text{normal}}\nZ = \{...,-3,-2,-1,0,1,2,3,... \} \leftarrow \frac{\text{real}}{\text{indegree}}
$$

$$
\frac{\frac{\frac{1}{\sqrt{9}}\text{ercsal way to describe a set}}{\frac{1}{9}}}{\frac{1}{9}\text{descciption}}{\frac{1}{9}\text{ermsat} + \frac{1}{9}\text{ermsat} + \frac{1}{9}\text{ermsat} + \frac{1}{9}\text{ermsat}}}
$$
\n
$$
\frac{\frac{\frac{1}{\sqrt{9}}\text{ercsal area}}{\frac{1}{9}\text{ermsat} + \frac{1}{9}\text{ermsat}}}{\frac{\frac{1}{9}\text{ermsat} + \frac{1}{9}\text{ermsat}}{\frac{1}{9}\text{ermsat}}}
$$
\n
$$
= \frac{1}{2} \frac{1}{7} \frac{2}{7} \frac{1}{2} \frac{1}{7} \frac{1}{2} \frac{1}{7} \frac{5}{7} \frac{1}{7} \frac{1}{7
$$



 $\mathbb{R} = \left\{ \times \; \middle| \; \times \; \text{has a decimal expansion} \right\}$ Set of  $=$   $\left\{\begin{array}{c} 1-\frac{5}{2} \\ 1-\frac{5}{2} \end{array}\right\}$   $\left\{\begin{array}{c} 1, 1, ... \\ 1, 0, ... \end{array}\right\}$ <br>
(1,0, -2.5 1.414...)<br>
3,14159 3, 14159.

Def: Let A and B be sets. Vef: Let A and D be sers.<br>We say that B is a subset of A, We say that  $B$ <br>and write  $B \subseteq A$ , if every element of B is also an element of A. ) if every elem<br>element of A.  $Sone$   $B$ people<br>Write Def: Let A and B be set<br>
We say that B is a subset<br>
and write  $B \subseteq A$ , it every<br>
of B is also an element of<br>
Some<br>
people<br>
Write<br>
BCA<br>
for<br>
subset We say that  $B$  is a subset of A<br>and write  $B \subseteq A$ , if every element<br>of  $B$  is also an element of A.<br> $A$ <br> $Sone  
people  
write  
BCA  
for  
subset$ for subset



$$
\frac{Ex: Let}{A = \{12n | n \in \mathbb{Z}\}}
$$
  
B = \{3n | k \in \mathbb{Z}\}

Then,

$$
A = \{ ..., 12(-3), 12(-2), 12(-1), 12(0),12(1), 12(2), 12(3), ... \}= \{ ..., -36, -24, -12, 0, 12, 24, 36, ... \}
$$

and  
\n
$$
\beta = \{ ..., 3(-3), 3(-2), 3(-1), 3(0),\n3(1), 3(2), 3(3),... \}
$$
\n
$$
= \{ ..., -9, -6, -3, 0, 3, 6, 9, ... \}.
$$
\n
$$
\begin{array}{r}\n\downarrow \\
\downarrow \\
\downarrow\n\end{array}
$$
\nLet's prove it formally.

Technique: To show that  $A \subseteq B$ one way is to pick some XEA and then derive that  $x \in B$ . Ex: Show that  $\{12n\}n\in\mathbb{Z}\} \subseteq \{3k|ke\mathbb{Z}\}$  $\begin{array}{lll} \underline{\text{prob}} & \text{if} & \text{if} & \text{if} & \text{if} & \text{if} \\ \underline{\text{left}} & \times & \text{if} & \text{if} & \text{if} & \text{if} & \text{if} \end{array} \end{array}$ proof: Then,  $x = 12n$  where  $n \in \mathbb{Z}$ . Hence,  $x = 3(4n).$ Let  $R = 4n$ .  $4\pi\epsilon\chi$  because<br>Su,  $x = 3k$ .  $\chi$  is closed.<br>Thus,  $x \in \{3k | k\epsilon\chi\}$ . Therefore,  $\{12n \mid n \in \mathbb{Z}\} \subseteq \{3k \mid k \in \mathbb{Z}\}$ 

Def: Let A and B be sets. The <u>union</u> of A and B is  $AUB = \{x \mid x \in A \text{ or } x \in B\}$  $A\bigwedge(\bigwedge)^{+}\bigwedge)^{5}$ The intersection of A and B is  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ ANB

 $\vdash x$ :  $A = \{ (2, 5, 9, 20, \frac{1}{3}, 2) \}$  $B = \{-16, 9, \frac{1}{3}, 4\}$  $AVB = \{I_{2},S,Y,20,\frac{1}{3},2,76,49\}$  $extr a B$ A stuff  $5t$ 

 $ANB = \{9, \frac{1}{3}\}$ 

$$
(Method to show that A=B)When A and B are sets0 Show that A=B(2) Show that B=A
$$



Prove that 
$$
ANB = 2bklll\in L
$$
.\n\nProot:  
\n $\boxed{E}$ : Let's show  $AND = \{6l|l\in L\}$ .  
\n $\boxed{E}$ : Let's show  $AND$ .  
\n $Pick \text{some } x \in NN$ .  
\nThen,  $x \in A$  and  $x \in B$ .  
\n $So, x = 2k$  and  $x = 3n$  where  $k, n$  are integers.

 $\int$  0  $\,$  $2k = 3n$ . Jo, C.<br>Thus, 3n is even. (  $z - 51$ <br>3n is even (because 3n is<br>3n is even (because 3n is We can't have a being odd since then 30 would be odd.  $(Codd * old = old)$ 

So n is even. Thus,  $n = 2m$  where  $m$  is an integer.  $S_{0}$ ,  $x = 3n = 3(2n) = 6m$  $S_{0}$ ,  $x \in \{6l | l \in \mathbb{Z}\}.$ Thus,  $ANDZ56l|lEZ3$ .  $[2]:$  Now let's show  $\{S\}\setminus\{E\}Z^2\subseteq A$  NB. Let  $x \in \{6l | l \in \mathbb{Z}\}.$ Then  $x = 6j$  where  $j \in \mathbb{Z}$ . Thus,  $x = 2(3) \in A$ .  $And, x = 3(2j) \in B$  $Sw, x \in A \cap B.$  $Thus, 262|2EZ^{2} \subseteq AND.$ 

Def: Let A and B be sets.

\nWe say that A and B are disjoint if 
$$
ABC = \emptyset
$$
 where  $\emptyset$  is the empty set.

\n
$$
EX: A = \{1, 2\}
$$
  $ADB = \emptyset$ 

\n
$$
B = \{3, 4\}
$$
  $S_{\rho}$   $A$  and B are always set.

\nand B be sets.

$$
EX: A = \{1, 2\} \qquad A \cap B = \n\begin{cases} 1 & \text{if } B = \text{if } B \\ 0 & \text{if } B = \{3, 4\} \end{cases}
$$

Def: Let A and B be sets.<br>
We say that A and B are <u>disjoint</u><br>
if  $A \cap B = \phi$  where  $\phi$  is the<br>
empty set.<br>  $B = \{3,4\}$  So, A and B<br>  $B = \{3,4\}$  So, A and B<br>
Def: Let A and B be sets.<br>
The <u>difference</u> of A and B is<br>
A B Def: Let A and B be sets. of <sup>A</sup> and <sup>B</sup> is The <u>difference</u>  $B = \frac{5}{2},$ <br>Let A<br>difference  $B A - B = \{x \mid x \in A \text{ and } x \notin B\}$  $\sum_{i=1}^{n}$ A<br>OSC  $A = \{1, 2\}$  AnB= $\frac{4}{5}$ <br>  $B = \{3, 4\}$  So, A and B<br>
Bef: Let A and B be sets.<br>
The difference of A and B is<br>
A B A-B =  $\{x \mid x \in A \text{ and } x \notin B\}$ <br>  $A = \frac{8}{x}$  A-B =  $\{x \mid x \in A \text{ and } x \notin B\}$  $\frac{1}{x}$  X | XEM mine "T"<br>read: "all x where<br> $\frac{real: "all x w here}$  $\mathsf B$ 

not in B" Notation: Some people write  $A \setminus B$  $for A-B.$ 

 $EX: A = \{1, 2, 3, 4, 5, 6, 7\}$  $B = \{8, 10, 11, 2, 5, 1\}$  $A-B=\{3,4,6,7\}$  $B$ 



$$
A = \{10, 11, 20\} = A
$$
  
\n $A = \{1,2,3,4,5,6,7\}$   
\n $A - A = \phi$ 

 $A - \{10, 11, 20\} = A$ <br>  $A = \{1, 2, 5, 4, 5, 6, 7\}$ <br>  $A - A = \oint a \frac{\text{nothing}}{\text{left}}$ <br>  $Let A, B, C be sets.$ <br>
Prove: If  $A \subseteq B$ , then  $A - C \subseteq B - C$ <br>
We want to show that  $A - C \subseteq B - C$ . Problem  $A - \{10, 11, 20\} = 1$ <br>  $A = \{1, 2, 3, 4, 5, 6, 7\}$ <br>  $A - A = \emptyset$  4 (nothing)<br>  $A - A = \emptyset$  4 (nothing)<br>  $A = \{1, 2, 3, 4, 5, 6, 7\}$ <br>  $A = \emptyset$ <br>  $A = \$  $HW$  problem<br>
Let A, B, C be sets.<br>
Prove: If  $A \subseteq B$ , then  $A - C \subseteq B - C$ <br>
Prove: If  $A \subseteq B$ , then  $A - C \subseteq B - C$  $HW problem\nLet A, B, C be sets\n
$$
P_{\text{Cave: I.}} + A \leq B, +
$$
\n
$$
P_{\text{Cave: I.}} + A \leq B
$$
\n
$$
P_{\text{Cone:}} + A \leq B
$$$  $+hat A-C\subseteq B-C$ . We want to show Let x c A - $\overline{\mathcal{C}}$  . Then  $x \in A$  and  $x \notin C$ . Then  $X \in A$  and  $A \subseteq B$ ,<br>Since  $X \in A$  and  $A \subseteq B$ , we know that XER.  $A = \{1, 2, 3, 4, 5, 6, 7\}$ <br>  $A = A = \emptyset$  explicit <br>  $A - A = \emptyset$  explicit <br>  $A - A = \emptyset$  explicit <br>  $A - A = \emptyset$  explicit <br>  $A = \{2, 3, 6\}$ <br>  $B = \{2, 4, 6, 7\}$ <br>  $B = \{2, 4, 7\}$ <br>  $B = \{2, 6, 7\}$ <br>  $B = \{2, 6, 7\}$ <br>  $B = \{2, 6, 7\}$ <br>  $B =$ Thus,  $x \in B$  and  $x \notin C$ .<br>Hence  $x \in B - C$ . Hence  $x \in B - C$ ,<br>Therefore,  $A - C \subseteq B - C$ 

$$
\frac{\text{Def: Let A be a set where}}{\text{Using a universal set (So, AcU)}}
$$
\nThen the complement of A with  
\n
$$
\frac{\text{To find the complement of A with U}}{\text{as per}}
$$
\n
$$
\frac{\text{Conplement of A with U}}{\text{Out A}}
$$
\n
$$
= \frac{\text{C}}{2} \times 1 \times \text{C} \text{ und } \times \notin A}.
$$

Ex: U = E 1 , 2 , 3 , 4 , 5, 6 , 7, 8 , 9 , 10, <sup>11</sup> , <sup>123</sup>  $A = 2, 4, 6, 8, 10, 123$  $A^C = U A = \{1, 3, 5, 7, 9,$ 114 Ex:<br>
U = {1,2,3,4,5,6,7,8,9, 10, 11,12}<br>
A = {2,4,6,8, 10, 12}<br>
A = {2,4,6,8, 10, 12}<br>
A = {1,3,5,7,9, 11}<br>
Theorem: (de Morgan's laws) : (de Morgan's -

Theorem  $|aws\rangle$ Let U be a vniversal set.<br>Let U be a vniversal set. et U be a Universal of U.<br>Let A and B be subsets of U. Then :  $\text{Then:}\n\begin{array}{c}\n\text{A} \\
\text{A} \\
\text{B} \\
\text{C}\n\end{array} = \text{A}^c \cap \text{B}^c \quad \text{A}^c \quad \text{B}^c \quad \text{B}^c \quad \text{B}^c \quad \text{C}$  $(2) (AND)^{c} = A^{c} \cup B^{c}$  $(4,4,11)$ <br>  $(2,4,11)$ <br>  $(3,6,11)$ <br>  $(4,4,11)$ <br>  $(5,6,11)$ <br>  $(6,6,11)$ <br>  $(7,6,11)$ <br>  $(8,6,11)$ <br>  $(1,6,11)$ <br>  $(1,6$ 

Proof:	
$Let's$ prove (I).	You can try (2).
$Let's$ show (AVB) $\subseteq$ A <sup>c</sup> AB <sup>c</sup> .	
$Let x \in (AVB)$ .	
$Then, x \in U$	AB
$canx \notin AUB$ .	
$so, x \in U$ and $x \notin A$ or $x \in B'$ is not not $x \in A$ or $x \in B'$ is not not $x \in V$ and $x \notin A$ and $x \notin B$ .	
$So, x \in V$ and $x \notin A$ and $x \notin B$ .	$2450$
$Thus, x \in A^c$	$7(P \text{ or } Q) \in \text{provided}$
$So, x \in A^c$	$7(P \text{ or } Q) \in \text{provided}$
$So, x \in A^c$	$7$ means not
$So, x \in A^c$	$7$ means not

21: Now let's show 
$$
h \in A^{B} \subseteq (A \cup B)^{C}
$$
.  
\nLet  $y \in A^{C} \cap B^{C}$ .  
\nSo,  $y \in A^{C}$  and  $y \in B^{C}$ .  
\nSo,  $y \in U$  and  $y \notin A$  and  $y \notin B$ .  
\nThus,  $y \in U$  and  $y \notin A \cup B$   
\n $S_{0}$ ,  $y \in (A \cup B)^{C}$ .  
\nThus,  $A^{C} \cap B^{C} \subseteq (A \cup B)^{C}$ .  
\nBy (c) and (d) we have  $(A \cup B)^{C} = A^{C} \cap B^{C}$ 

Another way $h$ prove:
$x \in (AUB)$
$iff \times E$ U and $x \notin AVB$
$iff \times E$ U and $x \notin A$ and $x \notin B$



Lef: Let <sup>A</sup> and <sup>B</sup> be sets. The Cartesian product of A and <sup>B</sup> is  $AXB = \left\{ (a,b) \mid a\in A \text{ and } b\in B \right\}$ Note : (a, Def: Let A and B be sets.<br>The Cartesian product of<br>A and B is<br> $AXB = \{(a,b) | a \in A \text{ and } b \in B\}$ <br>Mote:  $(a,b)$  is called an ordered<br>pair, order matters for  $(a,b)$ <br>People have proposed various set Note:<br>Dair pair .  $\lim_{\delta r \, \text{der} \text{ markets}}$  for  $(a, b)$ People have proposed various set definitions for (a, b) . A and B is<br>AXB = {  $(a,b)$  |  $a \in A$  and  $b \in B$ }<br>Mote:  $(a,b)$  is called an ordered<br>Pair, order matters for  $(a,b)$ <br>People have proposed various set<br>definitions for  $(a,b)$ . For example<br>one is  $(a,b) = \{a, \{a,b\}\}$ one is (a , b) =  $\{a, \{a, b\}\}\$ 

$$
\frac{E \times E}{Z} = \{...,^{-1}, -3, -2, -1, 0, 1, 2, 3, 4, ...}\n\nZ - \{0\} = \{...,^{-1}, -3, -2, -1, 1, 2, 3, 4, ...}\n\nLet  $S = Z \times (Z - \{0\})$ .  
\nThen,  
\n
$$
S = \{ (a, b) | a, b \in Z \text{ and } b \neq 0 \}
$$
\n
$$
= \{ (0, 2), (-1, 7), (3, -1), ... \}
$$
\n
$$
\frac{A}{\{x, a\}} = \{ (0, 3, 2, 1, 1, 2, 3, 1, ... \})
$$
$$

Ex: Let A, B, C be sets.

\nProve that 
$$
A \times (Bnc) = (A \times B) \cap (A \times C)
$$
.

\nProof:

\n
$$
\boxed{\le}
$$
: We first show that  $A \times (Bnc) = (A \times B) \cap (A \times C)$ .

\nLet  $x \in A \times (Bnc)$ :  
\nThen,  $x = (m, n)$  where  $m \in A$  and  $n \in C$ .

\nSince  $n \in B \cap C$  we know  $n \in B$  and  $n \in C$ .

\nThus,  $(m, n) \in A \times B$  since  $m \in A$  and  $n \in C$ .

\nand  $(m, n) \in A \times C$  since  $m \in A$  and  $n \in C$ .

\nHence,  $x = (m, n) \in (A \times C) \cap (A \times C)$ .

\nHence,  $x = (m, n) \in (A \times B) \cap (A \times C)$ .

\n[ $A \times B$ ]  $\cap (A \times C)$ :  
\n $(A \times B) \cap (A \times C)$ :  
\n $A \times B$  and  $y \in A \times C$ .

\nThen,  $y \in A \times B$  and  $y \in A \times C$ .

\nSo,  $y = (0, p)$  where  $\theta \in A$  and  $p \in C$ .



#f: Let <sup>A</sup> be <sup>a</sup> set. set Def: Let A be a set.<br>We define the <u>power set</u> of <sup>A</sup> to be the set of all subsets of <sup>A</sup> , that is

 $P(A) =$  $P(A) = \left\{ B \mid B \subseteq A \right\}$ <br>The set of all Power<br>Set of A B where  $B \subseteq A$ 

 $EX: A = \{1, 2\}$ Subsets of A)  $\frac{156123}{150}$ empty set is  $\phi$  $\begin{array}{|c|} \hline \text{empty set is} \\ \hline \text{0, subset of every} \\ \text{0, set} \\ \hline \text{set} \\ \hline \text{set}$ a subset EX:  $A = \{1, 2\}$ <br>
Subsets of A)<br>  $\phi$ <br>  $\{1\}$ <br>  $\{2\}$ <br>  $\{2\}$ <br>  $\{1, 2\}$ <br>  $\{3\}$ <br>  $\{5\}$ <br>  $\phi = \{6, 2\}$ <br>  $\phi = \{7, 2\}$ <br>  $\phi = \{1, 2\}$ <br>  $\phi = \$ of every  $\{2\}$  $MPSISEI$ <br>  $\alpha$  subset<br>
of every<br>
set<br>  $\phi = \{\begin{matrix} 1 \\ 51 \end{matrix} \}$ <br>  $SET measI$ maty set is<br>a subset<br>of every<br>set<br>set<br>commental<br>set means:<br>If x es, then!<br>Tf x es, then!  $\{1, 2\}$  |  $\frac{\sqrt{SUPE} \text{COMPENTARY}}{SCT}$  $ST$  means: Fx)If <sup>x</sup> ES, thenxet) 3<br>
- (empty set is<br>
a subset<br>
of every<br>
set<br>  $\phi = \frac{27}{100}$ <br>
subset<br>  $\phi = \frac{27}{100}$ <br>
set means:<br>  $\frac{4 \times (If x \in \phi, then x \in T)}{\phi \in T \text{ means:}}$  $P(A)$  = 4  $f_X(Tfxe\phi,$  there is<br> $f_X(Tfxe\phi,$  there is  $\left\{\n\begin{array}{c}\n2 \\
2 \\
1\n\end{array}\n\right\}$ <br>  $\left\{\n\begin{array}{c}\n2 \\
2\n\end{array}\n\right\}$  $(\forall x)(\text{If } x \in \varphi,$ then xet)  $= 2^2 = 2$  $P(A) = \{ \phi, \{1\}, \{2\}, \{1,2\} \}$ ↓  $2\}$ <br>  $y = 4$ <br>  $z^2 = 2^{|A|}$ <br>  $z = 2^2 = 2^{|A|}$  $mpty set is  
\na subset  
\nof every  
\nset  
\n $\phi = \{ \}$   
\nSDE commentary  
\n $\{x \text{ If } x \in S, \text{ then } x \in T\}$   
\n $\phi \in T \text{ means:}$   
\n $\{\forall x \text{ If } x \in \phi, \text{ then } x \in T\}$   
\n $\{2\}, \{1,2\}\}$$ 

 $EX: B = \{5, 2, 1\}$  $P(B) = \left\{ \begin{array}{c} \phi \\ \phi \end{array} \right.$  $\Big\{$   $\Big\}$ , 223, 254,  $35,29$ ,  $\left\{ \begin{array}{c} 2 \end{array}, \begin{array}{c} 1 \end{array} \right\}$  $\{5,1\}$ ,  $\{5,2,1\}$  $\{5,2\},\{2,1\}$ <br>  $\{5,1\},\{5,2,1\}$ <br>
Note:  $|P(B)| = 8 = 2^3 = 2^{|B|}$ 2  $\c3$ Ex:  $B = \{5, 2, 1\}$ <br>  $P(B) = \{4, \{1\}, \{2\}, \{5\}\}$ <br>  $\{5, 2\}, \{2, 1\}$ <br>  $\{5, 1\}, \{5, 2, 1\}$ <br>
Note:  $|P(B)| = 8 = 2^3 = 2^{181}$ <br>
Theorem: If S is finite,<br>
then  $|P(s)| = 2^{151}$  $2^{109}$ Theorem: If S is finite, Ex:  $B = \{5, 2, 1\}$ <br>  $P(B) = \{6, 1\}, \{2\}, \{5\}$ <br>  $\{5, 2\}, \{2, 1\}$ <br>  $\{5, 1\}, \{5, 2, 1\}$ <br>
Note:  $|P(B)| = 8 = 2^2 = 2^{181}$ <br>
Theorem: If S is finite,<br>
Then  $|P(s)| = 2^{151}$  $= 2^{15}$ 

The rem: Let <sup>A</sup> and <sup>B</sup> be sets. Then,  $A = B$ if and only if  $P(A) = P(B)$ . Proof: (Fi)) It's clear that if then P(A) Theorem: Let A and B<br>be sets. Then,  $A = B$ <br>if and only if  $P(A) = P(B$ <br>proot:<br> $\Rightarrow P(B)$  It's clear that if<br> $A = B$ , then  $P(A) = P(B)$ .<br> $\Rightarrow P(B) = P(B)$ , then  $A = B$  $H's$  clear that it<br>B, then  $P(A) = P(B)$ . (3) Now we must prove  $U_{II} \circ (A) = P(B)$  $fhen$   $A = B$ ".  $Suppose P(A) = P(B).$ To show that  $A = B$  we

will show 
$$
A \subseteq B
$$
 and  $B \subseteq A$ .  
\n
$$
\boxed{\begin{array}{l}\n\text{(Liam 1: } A \subseteq B) \\
\text{We know } A \subseteq A.\n\end{array}}
$$
\n
$$
\begin{array}{l}\n\text{Then, since } P(A) = P(B), \\
\text{we know } A \in P(B).\n\end{array}}
$$
\n
$$
\begin{array}{l}\n\text{Thus, } A \subseteq B.\n\end{array}
$$
\n
$$
\begin{array}{l}\n\text{(Laim 2: } B \subseteq A) \\
\text{(Laim 2: } B \subseteq A) \\
\text{(Laim 3: } B \subseteq A) \\
\text{(Lam 4: } B \cup B) = \text{(Lum 5: } B \cup B) \\
\text{(Leth 6: } B \cup B) = \text{(Leth 7): } B \cup B = \text{(Leth 8): } B \cup B = \text{(Leth 8): } B \cup B = \text{(Leth 9): } B \cup B = \text{(Leth 1): } B \cup B = \text{(Leth 2): } B \cup B = \text{(Leth 3): } B \cup B = \text{(Leth 4): } B \cup B = \text{(Leth 5): } B \cup B = \text{(Leth 6): } B \cup B = \text{(Leth 7): } B \cup B = \text{(Leth 8): } B \cup B = \text{(Leth 9): } B \cup B = \text{(Leth 1): } B \cup B = \text{(Leth 2): } B \cup B = \text{(Leth 1): } B \cup B = \text{(Leth 3): } B \cup B = \text{(Leth 2): } B \cup B = \
$$

Thus,  $\{b\} \subseteq A$ . Hence b E A.  $S_0, B \subseteq A$ .

 $By claim 1 and 2 we  
know A=B.$ 



Def: When every<br>of a set A is Def: When every element Vet: When every element<br>of a set A is itself a set then We call A a family or <u>collection</u> Def: When every element<br>of a set A is itself a<br>set then we call A<br>a fumily or <u>collection</u> of sets<br>Ex:  $P(A)$  is a family<br>of sets. of sets. Def: When every element<br>of a set A is itself a<br>set then we call A<br>a <u>family</u> or <u>collection</u> of se<br>Ex: P(A) is a family<br>of sets.

 $EX: P(A)$  is a family of sets .

non-emply Vet: Let A be a family of sets. Define the <u>union over</u> A  $+$  be  $\bigcup_{i} S = \left\{ x \mid x \in S \text{ for some } S \in A \right\}$  $=\left\{ x \mid \begin{matrix} \text{there exists some} \\ \text{S} \in A \end{matrix} \right\}$ SEA  $Ex:$  $A = 55, 52, 53, 54$ Shaded  $\int$ SEA

Define the intensection over A  $\n *i*$  $\bigcap S = \{x \mid x \in S \text{ for all } S \in A\}$ SEA  $A = 55, 52, 534$ Shaded blue is  $56A$ 

$$
E_X: \{2, 4, 6, 8\}, \{1, 3, 2\},
$$
\n
$$
\{2, 7, 6\}, \{3, 2\},
$$
\n
$$
\{3, 3, 2\},
$$
\n
$$
\{4, 8, 3, 1\},
$$
\n
$$
\{4, 8, 2, 3, 4, 6, 3, 1\},
$$
\n
$$
\{5, 2, 3, 4, 6, 3, 5, 2\},
$$
\n
$$
\{6, 2, 3, 1\},
$$
\n
$$
\{1, 5, 5, 2, 3, 4, 6, 3, 8\}
$$
\n
$$
S_1: \{1, 2, 3, 4, 6, 3, 8\}
$$
\n
$$
S_2: \{2, 3, 4, 6, 3, 8\}
$$
\n
$$
S_3: \{2\}
$$
\n
$$
S_4: \{2, 3, 4, 6, 3, 8\}
$$

 $EXI N = \{1, 2, 3, 4, ...\}$  $Z,\{\cdots, -2, -1, 0, 1, 2, ...\}$  $B=\begin{cases} \sum n\in\mathbb{Z} \mid n\leq k\end{cases}$  $=\{S_{k} | k \in \mathbb{N}\}\$ <br>=  $\{S_{1}, S_{2}, S_{3}, S_{4}, ... \}$ and  $S_k = \{ n \in \mathbb{Z} \mid |n| \leq k \}$  $S = \{ n \in \mathbb{Z} \mid |n| \leq |\} = \{-1, 0, 1\}$  $S_{2} = \{ n \in \mathbb{Z} \mid |n| \leq 2 \} = \{ -2, -1, 0, 1, 2 \}$  $S_3 = \{-3,-2,-1,0,1,2,3\}$ 

 $S_{4} = 2 - 4$ ,  $\mathcal{F}_\mathcal{G}$  $-2,$ - 1, 0, 1, 2, 3, 4 } And w so on...  $US = 2$  $\frac{1}{\infty}$  $S_{k}$  is another way<br>Sk is another way<br>Sk is another way<br>Sk is another way is another way  $S_{4} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ <br>And so on...<br> $S = \mathbb{Z}$   $(S_{k}$  is another way<br> $S \in \mathbb{B}$  $I = \{-4, -3, -10\}$  $2, -1, 0, 1, 2, 3, 4$ is another way<br>to write it  $\bigcap$  S =  $\left\{ -1, 0 \right\}$  $\frac{k-1}{k-1}$  to write it<br>  $y11$ <br>  $x=1$  Sk is another way SEB

Def: Let 1 be a non-empty Set. Suppose for each  $\alpha \in \mathbb{I}$ there is a corresponding set  $A_{\alpha}$ . The tamily  $A = \{A_\alpha \mid \alpha \in I\}$ is called an <u>indexed family</u> of sets. The set I is called the <u>index set</u>. If  $d\in I$ , then a is called the index of Ax.

Ex: Previously we had  $B = \left\{S_{k} \mid k \in \mathbb{N}\right\}$ Here B is an indexed family re B is an indexed taming<br>of sets. N is the index set.  $B = \begin{cases} S_{k} \mid k \in \mathbb{N} \end{cases}$ <br>tere  $B$  is an indexed family<br>of sets. N is the index set.<br> $S_3 \leftarrow \underbrace{\alpha = 3 \text{ is the index of } S_3}$  $36453$  is the index of  $53$ 

Det: Let I be a non-empty Set. Let  $A = \{A_{\alpha} | \alpha \in I\}$ he an indexed family of sets. Define the <u>union over A</u>  $\bigcup_{\alpha\in I}A_{\alpha}=\left\{x \mid \begin{matrix}there exists \ \alpha\in I \\ \text{with } x\in A_{\alpha} \end{matrix}\right\}$  $\bigcap_{\alpha\in I}A_{\alpha}=\left\{ x\mid\begin{array}{c} x\in A_{\alpha}\text{ for }\\ \alpha\text{ if } \alpha\in I\end{array}\right\}$  $E_{X}: \int In \urcorner \text{Our } P(e \urcorner) \text{us } c \arccos \text{or} P le \urcorner$ we would write  $US_{k} = \mathbb{Z}$  and  $NS_{k} = \{2, 0, 1\}$ <br>REN

[Ex:] Let's make sense of  $\bigcup_{\alpha\in\mathbb{Z}} (\alpha, \alpha+1)$  and  $\bigcap_{\alpha\in\mathbb{Z}} (\alpha, \alpha+1)$  $dE$ Where  $(d,d+1)$  means the interval in the rcal numbers R  $\Rightarrow$  IK  $\frac{1}{\alpha}$   $\frac{1}{\alpha}$   $\frac{1}{\alpha}$   $\frac{1}{\alpha}$   $\frac{1}{\alpha}$  $T = U$  $A_{\alpha}=(\alpha,\alpha+1)$  $A = \{A_\alpha | \alpha \in \mathbb{Z}\}$ 

A-4 A-3 A<sub>-2</sub> A<sub>-1</sub> Ao A1 A2 A3 A4<br><del>(IIIII)(IIII)(IIII)(IIII)(IIII)(IIIII)(IIII)(IIII)</del><br>-3 -2 -1 0 1 2 3 4 R

$$
U A_{\alpha} = R - Z
$$
  
\n $\alpha E Z$   
\n $\alpha nO + her$  way to write this is  $U A_{\alpha}$   
\n $W A_{\alpha} = -\infty$   
\n $W B_{\alpha} = -\infty$   
\n $W C_{\alpha} = -\infty$ 

$$
\underbrace{\bigcap_{\alpha \in \mathbb{Z}} A_{\alpha}}_{\text{unobt and way to write}} \bigcap_{\alpha = -\infty}^{\infty} A_{\alpha}
$$
\n
$$
\underbrace{\bigcap_{\alpha \in \mathbb{Z}} A_{\alpha}}_{\text{which numbers}} \bigcap_{\alpha \in -\infty} A_{\alpha}
$$

Theorem: Let A={Ax|dEI} be an indexed family of sets. Let  $\alpha_0 \in I$  be a fixed element. Then:<br>  $0 A_{\alpha_{0}} \subseteq \bigcup_{\alpha \in I} A_{\alpha} \overline{I_{\alpha_{0} = S}^{E_{\alpha}}}$ <br>  $0 A_{\alpha_{0}} \subseteq A_{\alpha_{0}}$ <br>  $0 A_{\alpha} \subseteq A_{\alpha_{0}}$ <br>  $0 A_{\alpha} \subseteq A_{\alpha_{0}}$ <br>  $0 A_{\alpha} \subseteq A_{\beta}$ proof: O Pick some  $x \in A_{\alpha_{s}}$ <br>Then there exists  $\alpha \in I$  (namely)<br>Where  $x \in A_{\alpha}$ 

 $Jhuf$  $X \in \bigcup_{\alpha \in \mathcal{I}} A_{\alpha} = \left\{ y \mid \begin{matrix} \text{there exists} \\ \text{dET with} \\ \text{y} \in A_{\alpha} \end{matrix} \right\}$ 



 $(2)$  You try.